Geometrical clusterisation and phase transition in SU(2) gluodynamics

O. Ivanytskyi

Bogolyubov Institute for Theoretical Physics, Kyiv, Ukraine

(with **K. Bugaev**, **E. Nikonov**, **E.-M. Ilgenfritz**, **D. Oliinychenko**, **V. Sagun**, **I. Mishustin**, **V. Petrov** and **G. M. Zinovjev**)

Nucl. Phys. A 960 (2017) 90-113

Dubna, 11 July 2017

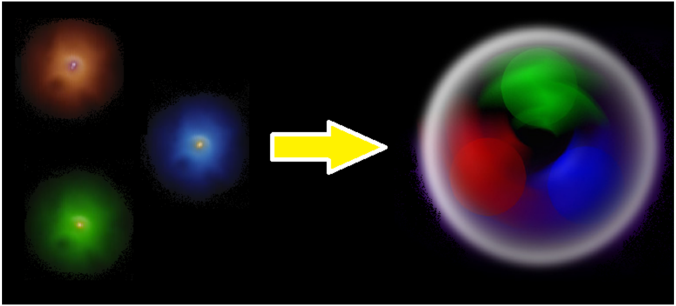
Outline

Outline

Outline

Motivation

- Therory of strong interactions (QCD) operates with quarks and gluons
- Only hadrons are directly observed in experiments



Clusterization of partons to hadrons

Motivation

- Break down/restoration of Z(N) center symmetry $\Rightarrow$ deconfinement phase transition in SU(N) gluodynamics

- Symmetry language is proven to be successful in studies of the liquid-solid, solid-solid and magnetic phase transitions

- Deconfinement in QC_3D (probably) is a phase transition of the liquid-gas type
  - hadron matter at low densities and temperatures is a gas
  - QGP (probably) is the most perfect liquid
    (hydrodynamic expansion, small η/s, typical value of $\frac{U_{potential}}{T}$)
    E. Shuryak, Prog. Part. Nucl. Phys. 62, 48-101 (2009)

Is phase transition in gluodynamics related to the liquid-gas one?

One more reason to look for possible new approaches

- Exceptional role of QCD order parameters ($\langle\overline{\psi}\psi\rangle$ and $\langle L\rangle$) is washed out in presence of massive quarks

New order parameters are of interest?

Svetitsky -Jaffe conjecture

- Deconfinement transitions in (d+1)dimensional SU(N) gluodynamics is equivalent to magnetic transition in the d-dimensional Z(N) spin system
  L. G. Yaffe and B. Svetitsky, PRD, 26, 963, 1982

$$SU(2)\ gluodynamics \ \Leftrightarrow \ Ising\ spin\ model$$

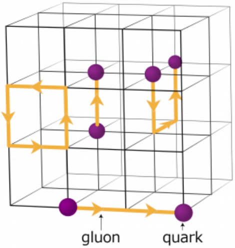
- Local Polyakov loop - gauge invariant analog of continuous spin

$$L(\vec{x}) = Tr \prod_{t=0}^{N_\tau - 1} U_4(\vec{x}, t)$$

$$U_4(\vec{x}, t) - temporal\ gauge\ link$$

$$defined\ by\ gluon\ field$$

$$SU(2) \ \Rightarrow \ L(\vec{x}) \in [-1, 1],\ real$$



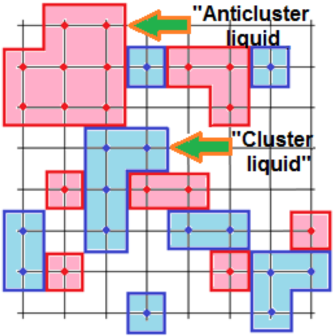
Identification of geometrical clusters

Definition of (anti)clusters

$$|L(\vec{x})| < L_{cut} \Rightarrow auxiliary\ vacuum$$
$$|L(\vec{x})| \geq L_{cut} \Rightarrow (anti)clusters$$
$$L_{cut} - vacuum\ cut - off\ \ parameter$$

C. Gattringer, PLB, 690, 179 (2010)

C. Gattringer, A. Schmidt, JHEP 1101, 051, 2011

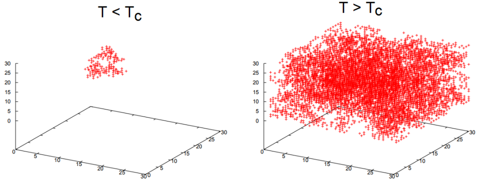


(Anti)clusters can be either “spin up"or “spin down"ones

- Largest fragment - “anticluster liquid droplet"
- Next to the largest fragment - “cluster liquid droplet"
- Gas of (anti)clusters has the same Polykov loop sign as their “liquids"

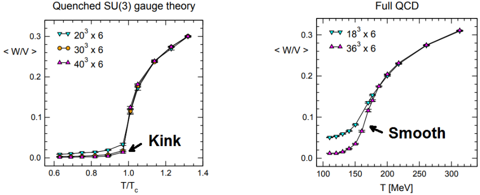
Percolation in gluodynamics

- Properties of the largest cluster are studied in SU(2), SU(3) and SU(4) GD



S. Borsanyi, J. Danzer, Z. Fodor, C. Gattringer, A. Schmidt, J. Phys. Conf. Ser. 312, 012005, 2011

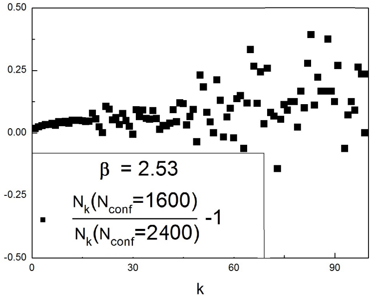
- Statistical weight of the largest cluster is smooth in presence of quarks



How to recognize phase transition?
Can small clusters help?

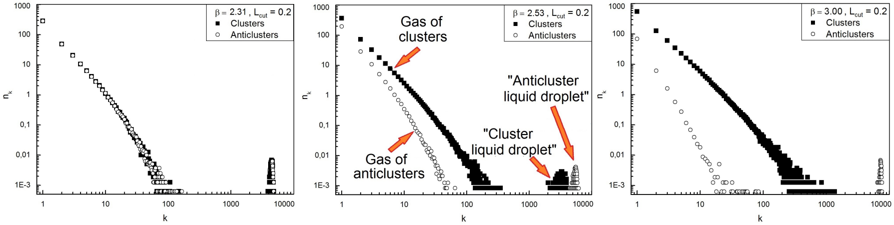
Numerical simulation of distributions

- $3 + 1$ dimensional lattice of size $N_\sigma = 24$, $N_\tau = 8$
- 13 values of inverse coupling $\beta \in [2.31, 3]$ $\Rightarrow$ 13 values of physical temperature
- vacuum cut-off parameter $L_{cut} = 0.1$ and 0.2
- Average over 800, 1600 and 2400 independent configurations for all β and L_{cut}



Saturation of distributions $\Rightarrow$ $N_{conf} = 2400$ is taken as the high statistics limit

Size distributions of (anti)clusters



Distributions at low $\beta \leq \beta_c \simeq 2.52$ (phase of restored global Z(2) symmetry)

- symmetry between (anti)cluster distributions
- gas and "liquid"domains are well separated

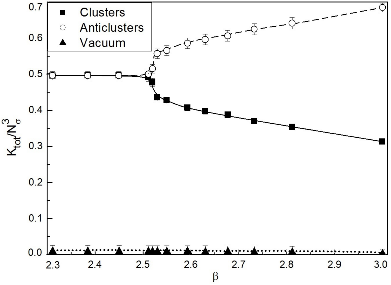
Distributions at high $\beta > \beta_c \simeq 2.52$ (phase of broken global Z(2) symmetry)

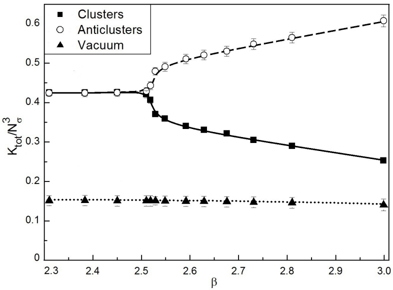
- no symmetry between (anti)cluster distributions
- "cluster liquid"evaporates to cluster gas
- anticluster gas condensates to "anticluster liquid"

Deconfinement phase transition (at least in SU(2) GD)
= special kind of the liquid-gas phase transition?

Volume fraction

$$K_{tot} = \begin{cases} \sum_k k n_k^{(a)Cl} \Big/ N_\sigma^3, & (anti)clusters \\ 1 - K_{tot}^{aCl} - K_{tot}^{Cl}, & auxiliary\ \ vacuum \end{cases}$$



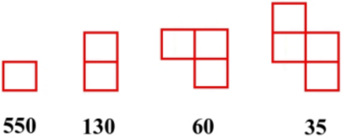


Volume fraction of vacuum is independent on β and/ore temperature

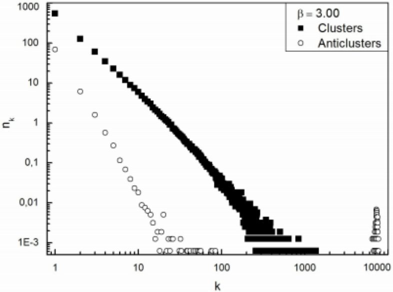
Incompressible auxiliary vacuum?

Space inhomogeneity

$^{max}K_{acl}$ = 7300 - the volume of largest anticluster;
$^{max}K_{cl}$ = 223 - the volume of largest cluster;
V^{gas}_{acl} = 89 - the total volume of the gas of anticlusters;
V^{gas}_{cl} = 2848 - the total volume of the gas of clusters;
V_{vac} = 1707 - the volume of auxiliary vacuum;



$\beta = 3.0, \; L_{cut} = 0.2$



$$N^{near}_{cl} \simeq 6\, n_{cl}(1) + 10\,(n_{cl}(2) + n_{cl}(3) + n_{cl}(4)) \simeq 5600.$$

Visually these liquids resemble two pieces of different Swiss cheeses!

Liquid droplet approach

- Deconfinement transition in SU(2) gluodynamics is a special kind of the liquid-gas phase transition with two liquids and two gases

Polyakov loop clusters ⇔ droplets?

- Liquid droplet formula for average number of (anti)clusters of size k

first introduced in M.E. Fisher, Physics 3, 255 (1967)

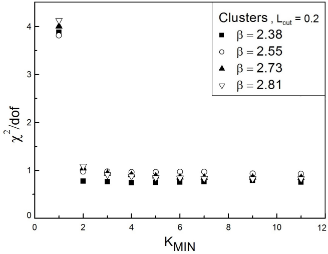
$$n_{k \geq k_{min}} = C \exp\left(\nu k - \sigma k^{2/3} - \tau \ln k\right)$$

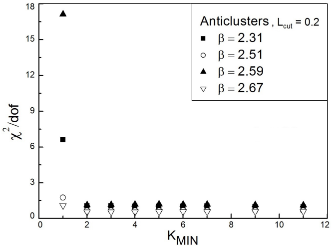
- C - normalization factor (absolute amount)
- ν - reduced chemical potential (liquid-gas phase transition)
- σ - reduced surface tension coefficient (appearance of critical point)
- τ - Fisher topological exponent
  (size distribution at critical point and critical exponents)
- k_{min} - size of the minimal (anti)cluster described by the liquid droplet formula

Applicability of the Liquid Droplet formula

- Too small (anti)clusters can not be treated as droplets
  What is minimal size of (anti)clusters which are described by the LDF?

- Overall χ^2/dof calculated for distributions of clusters with size $k \geq k_{min}$





- LDF describes size distributions with almost the same quality for all $k_{min} \geq 2$

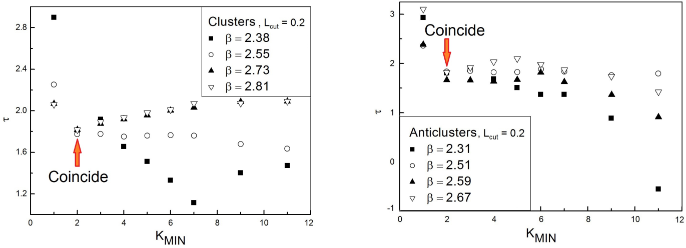
How to define k_{min}?

Determination of k_{min} and τ

- Fisher topological exponent τ is supposed to be constant

  M.E. Fisher, Physics 3, 255 (1967)

- Fit of distributions of clusters with size $k \geq k_{min}$

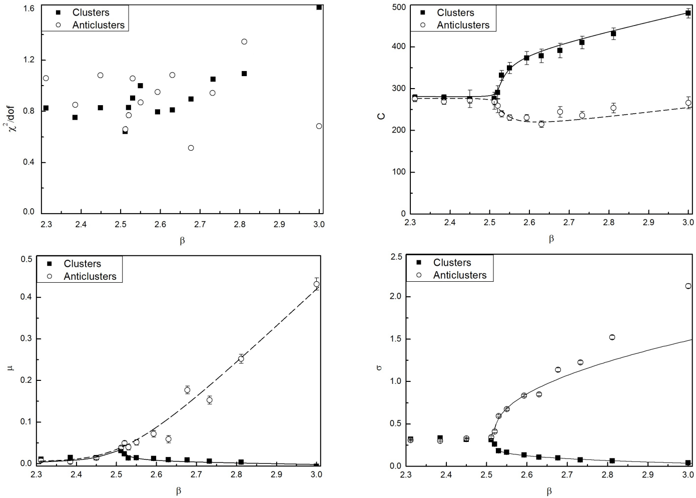


- $\tau = 1.806 \pm 0.008$ is independent on temperature at $k_{min} = 2$
  Found value of τ agrees with exactly solvable statistical models for:
  - nuclear matter

    P. T. Reuter, K. A. Bugaev, Phys. Let. B 517, 233 (2001); Ukr. J. Phys. 52, 489 (2007).
  - tricritcal endpoint of QCD

    OI and and K. A. Bugaev, Ukr. J. Phys. 57, (2012) 964

Reduced chemical potential and surface tension



At $\beta = 2.52$ global Z(2) symmetry breaks down $\Rightarrow$
chemical nonequilibrium between (anti)clusters ($\nu_{cl} \neq \nu_{acl}$)

Properties of monomers

Idea: introduce effective volume and surface of monomers and fit their multiplicities!

A priori we do not know, if the Liquid Gas Model formula can work…

Include the monomers into fit

$$\chi_A^2 = \sum_{i=1}^{N_\beta} \left(\underbrace{\frac{\left[{}_A n_{k=1}^{th} - {}_A n_{k=1}\right]^2}{[\delta_A n_{k=1}]^2}}_{monomers} + \sum_{k=2}^{k_{max}(\beta)} \frac{\left[{}_A n^{th} - {}_A n\right]^2}{[\delta_A n]^2} \right),$$

$$dof = N_\beta - 3 + \sum_{i=1}^{N_\beta} (k_{max} - 2 - 3) = \sum_{i=1}^{N_\beta} k_{max} - 4N_\beta - 3$$

i counts for all β values: N_β is their number

Properties of monomers

Parameterization of monomer multiplicity with effective volume and effective surface

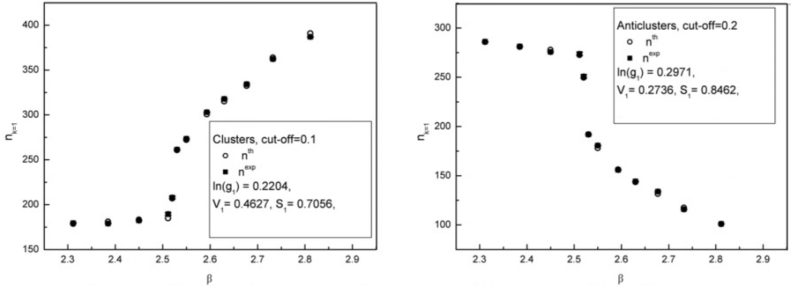
Assume $$ln\left({}_A n^{th}_{k=1}\right) = ln\, C_A + \underbrace{ln\, g_A}_{correction} + \underbrace{\mu_A \cdot V_A}_{effective\ V} - \underbrace{\sigma_A \cdot S_A}_{effective\ S}$$

parameters C_A, μ_A and σ_A are taken from k-mers $(k > 1)$

g_A, V_A and S_A can be found by the maximum-likelihood method as

$$\frac{\partial \chi^2_A}{\partial\,(ln\ g_A)} = 0\,, \qquad \frac{\partial \chi^2_A}{\partial V_A} = 0\,, \qquad \frac{\partial \chi^2_A}{\partial S_A} = 0\,.$$

Properties of monomers



data	*cut − off*	$\ln g_A$	V_A	S_A	$\frac{S_A}{\ln g_A}$	$\frac{\chi^2_A}{dof}$
Anticlusters	0.1	0.30578±0.00103	0.42274±0.00927	0.85643±0.00099	2.80083	1.4685
Clusters	0.1	0.22039±0.00068	0.46265±0.04176	0.70556±0.00179	3.20143	0.8952
Anticlusters	0.2	0.29712±0.00081	0.27362±0.00952	0.84625±0.00115	2.84820	0.9567
Clusters	0.2	0.17568±0.00056	0.99028±0.05024	0.52508±0.00288	2.98878	0.9173

Total fit quality is good! This ratio is puzzling!

Properties of monomers

Effective Surface Tension of Monomers

g_A is nearly the same for both cut-offs and for both types of clusters ($\pm 7\%$)

It seems that the correction of normalization g_A is redundant!

Then one can get rid of it

$$ln\left({}_A n^{th}_{k=1}\right) \simeq \ln\ C_A + \mu_A V_A - S_A\left(\sigma_A - \frac{1}{3}\right)$$

effective surface tension

For monomer clusters effective surface tension is negative
at \beta = 1.3 \beta_c for cut-off = 0.1
at \beta = \beta_c for cut-off = 0.2

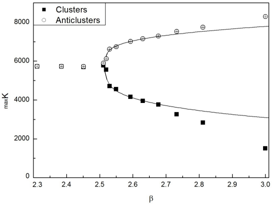
For monomer anticlusters the effective surface tension > 0 always!

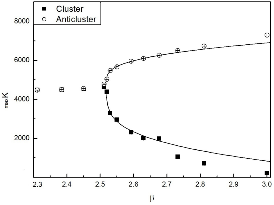
Average maximal (anti)cluster

- **Average Polyakov loop** is SU(2) gluodynamics order parameter, not observable
- **Largest (anti)cluster** occupies almost all lattice $\Rightarrow |L| \sim \max K_{aCl} - \max K_{Cl}$

$$\max K = \sum_{\vec{x}} k^{1+\tau} n_k \Big/ \sum_{\vec{x}} k^{\tau} n_k$$

$$\beta > \beta_c: \ \max K(\beta) - \max K(\beta_c) = a \cdot (\beta_c - \beta)^b$$



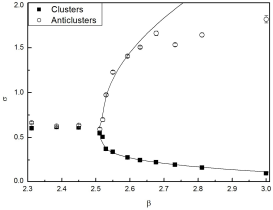


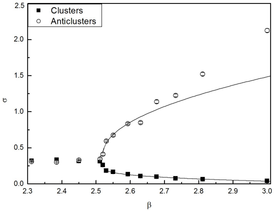
L_{cut}	type	a	b	χ^2/dof
0.1	Cl	-3056 ± 246	0.2964 ± 0.0284	$16.32/4 \simeq 4.08$
0.1	aCl	2129 ± 160	0.3315 ± 0.0269	$8.94/4 \simeq 2.235$
0.2	Cl	-4953 ± 443	0.3359 ± 0.0289	$12.3/3 \simeq 4.01$
0.2	aCl	2462 ± 87.7	0.3750 ± 0.0129	$2.068/4 \simeq 0.517$

Exponent b coinside with b_{Ising} of the Ising model - Svetitsky-Jaffe conjecture

Reduced surface tension coefficient

$$\beta > \beta_c : \ \sigma(\beta) - \sigma(\beta_c) = d \cdot (\beta_c - \beta)^B$$





L_{cut}	type	d	B	χ^2/dof
0.1	Cl	-0.485 ± 0.014	0.2920 ± 0.0012	$1.43/4 \simeq 0.36$
0.1	aCl	2.059 ± 0.028	0.4129 ± 0.0077	$1.68/4 \simeq 0.48$
0.2	Cl	-0.2796 ± 0.0118	0.2891 ± 0.0016	$1.11/4 \simeq 0.28$
0.2	aCl	1.344 ± 0.033	0.4483 ± 0.0021	$0.66/2 \simeq 0.33$

$$|L| \sim \max K_{aCl} - \max K_{Cl} \sim (\sigma_{aCl} - \sigma_{Cl})^{b/B}$$

Reduced surface tension coefficient - order parameter

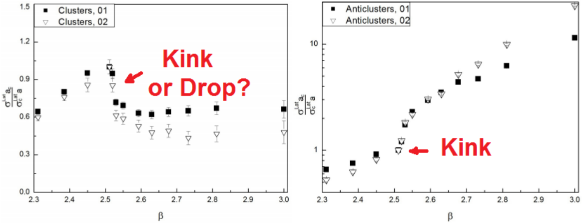
Physical surface tension surface tension

- Reduced surface tension σ enters LDF as part of the Boltzmann exponential

$$n_k \sim \exp(\ldots - \sigma k^{2/3} - \ldots), \quad \sigma k^{2/3} = \frac{\sigma_{phys}(a^3 k)^{2/3}}{T}$$

a – temperature dependent lattice spacing

- Physical Surface tension



- Surface tension of gluonic tube is related to tension of color string

K. Bugaev et al., Yad. Fys. 75, 6, 757 -759 (2012)

see talk of **V. Braguta** on σ_{str} in QC_2D

Irregularity of surface tension is not washed out in presence of quarks?

New order parameter?

Conclusion

- The approach to study the properties of the Polyakov loop geometrical (anti)clusters is developed
- It is shown that the deconfinement phase transition can be explained by the condensation/evaporation of large anticluster/cluster "liquid droplet which corresponds to Z(2) global symmetry breaking
- The size distributions of the gas of (anti)clusters are analyzed on the basis of the Liquid Droplet Model It is shown that even dimers are described within this approach with high accuracy
- The Fisher topological constant τ is found to be 1.806 ± 0.008
- It is shown that the reduced surface tension of (anti)clusters can serve as an order parameter which is able to distinguish the phases of restored and broken Z(2) global symmetry
- Need model: Svetitsky-Jaffe inspired statistical models, modified models with liquid-gas phase transition, instanton-dyon model for gluodynamics ...


THANK YOU
FOR YOUR ATTENTION